



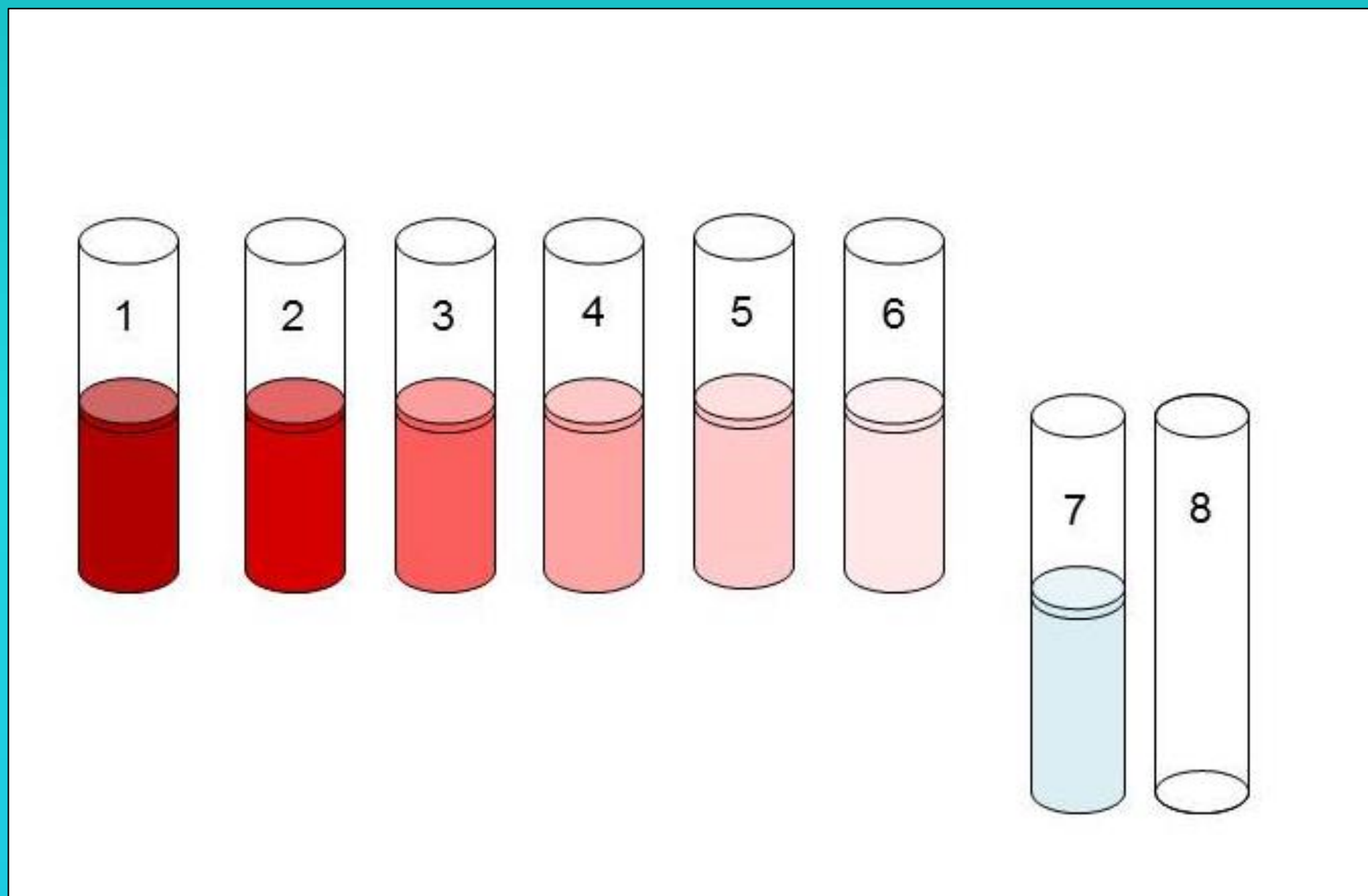
# Weber-Fechner's law, the Goldman-Hodgkin-Katz equation and curve fitting:

## a hands-on experiment for the undergraduate laboratory

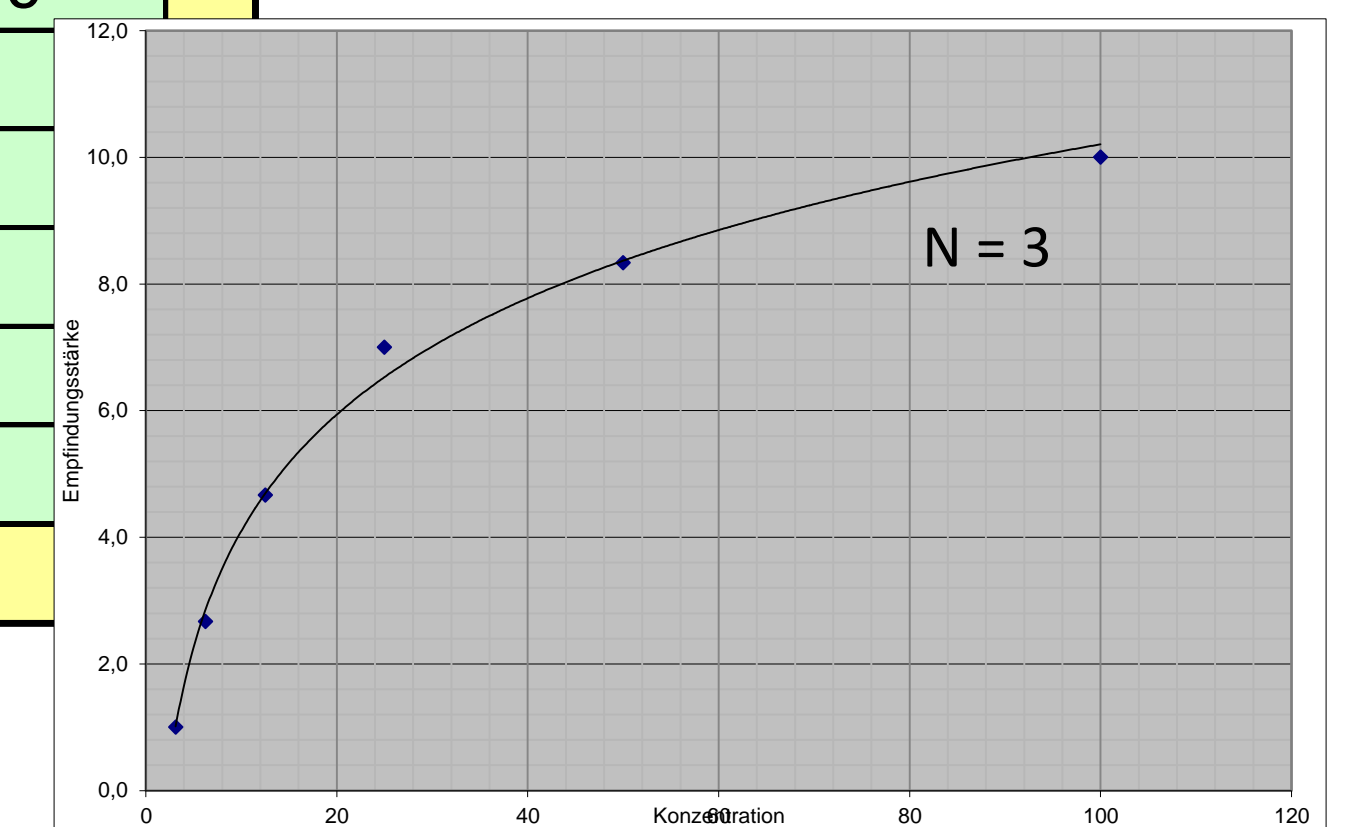
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### Introduction & Methods:

Weber-Fechner's law is an impressive example of early attempts to study the human response to a physical stimulus in a quantitative fashion. We attempted to develop an experiment avoiding the pitfalls of adaptational responses and suitable for training undergraduate students in data analysis using standard software.



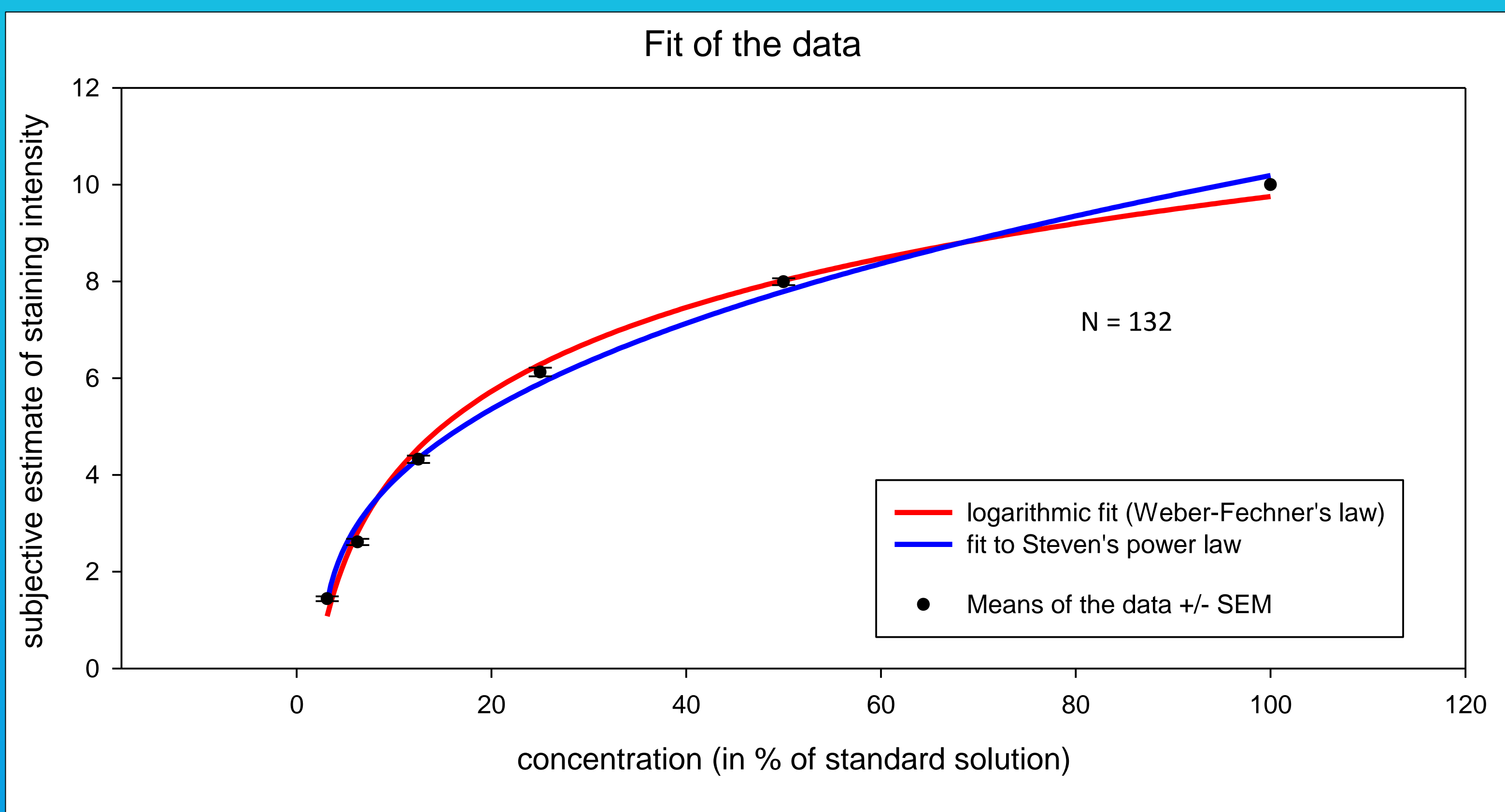
Röhrchen	Konzentration (%)	Farbempfindungswert			
		Proband 1	Proband 2	Proband 3	Mittelwert
1	100	10	10	10	10.0
2	50				
3	25				
4	12.5				
5	6.25				
6	3.125				



132 undergraduate students working in groups of 2 were given a non-toxic red stock solution (e.g. cherry juice, 100%) and water (0%) and asked to prepare samples of 50%, 25%, 12.5%, 6.25 and 3.125% concentration ("x") via serial dilution.

The two students in the group and one uninvolved test person then ranked the staining intensity of the solutions on a subjective linear scale ("Ψ(x)") ranging from 10 (pure stock solution, bright red) to 0 (pure water). The concentrations "x" of the solutions and the subjective estimates of staining intensity "Ψ(x)" were entered into a pre-prepared excel worksheet. Instructions were given on how to calculate the mean, plot the results (n=3), and fit the data with a logarithmic function.

### Results



In all but one of the 66 groups, a logarithmic fit of Ψ(x) versus x yielded a coefficient of determination R<sup>2</sup> > 0.90, in 58 groups, R<sup>2</sup> was > 0.97. A fit of the means of all 132 measurements to Weber-Fechner's law gave

$$\Psi(x) = 2.5 \cdot \ln(x) - 1.77 \quad (R^2 = 0.99).$$

In principle, the data could also be fitted to Steven's power law

$$\Psi(x) = 1.85 \cdot (x - 2.68)^{0.37} \quad (R^2 = 0.99)$$

However, this requires introduction of a third constant, which is not offered by the basic trendlines menu of excel. Since the more complex fitting routines of excel tend to be difficult to handle, it is not a good idea to attempt such a fit in an undergraduate course. More advanced students should probably be introduced to software specialized for curvefitting (e.g., Igor Pro, SigmaPlot).

### Discussion

**Lineare Zunahme der Na Permeabilität mit der Reizstärke:**

$\Delta p(Na)/Reiz = Reizantwort \cdot Reizstärke \cdot \phi$

Reizantwort = 0.01

**Basispermeabilitäten**

$p_{Na} = 0.01$   
 $p_{K} = 1$   
 $p_{Cl} = 0.01$

**Ionenkonzentrationen**

**intrazellulär**  
[Na]<sub>i</sub> = 8  
[K]<sub>i</sub> = 130  
[Cl]<sub>i</sub> = 10

**extrazellulär**  
[Na]<sub>o</sub> = 140  
[K]<sub>o</sub> = 4  
[Cl]<sub>o</sub> = 110

**Rezeptormodell (Goldman-Hodgkin-Katz)**

Reizstärke ϕ

Rezeptorpotential (mV)

$y = 15.107 \ln(x) - 84.655$

Rezeptorpotential (mV) — Log. (Rezeptorpotential (mV))

$Rezeptorpotential = 61.5 mV \cdot \log \left( \frac{pNa(Reiz) \cdot [Na]_i + pK \cdot [K]_i + pCl \cdot [Cl]_o}{pNa(Reiz) \cdot [Na]_o + pK \cdot [K]_o + pCl \cdot [Cl]_i} \right)$

**z. B. Duftmoleküle**

offene Ionenkanäle

Reizstärke ϕ	Δ p(Na) <sub>Reiz</sub>	Rezeptorpotential (mV)
1	0.02	-78.68
2	0.03	-73.76
3	0.04	-69.61
4	0.05	-66.03
5	0.06	-62.87
6	0.07	-60.05
7	0.08	-57.50
8	0.09	-55.18
9	0.10	-53.05
10	0.11	-51.07
11	0.12	-49.24
12	0.13	-47.52
13	0.14	-45.91
14	0.15	-44.40
15	0.16	-42.96
16	0.17	-41.60
17	0.18	-40.31
18	0.19	-39.08
19	0.20	-37.91
20	0.21	-36.78

A model for the receptor potential based on the Goldman-Hodgkin-Katz equation is developed on a separate spreadsheet. The limits of this model and of other models of sensory perception are discussed.

### Modell Schmerzwahrnehmung

Reizintensität	Schmerzwahrnehmung
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331

Werte für die Reizintensität in die weißen Felder eingeben!

Substanz P und andere Botenstoffe: erregen weitere Nozizeptoren!

In particular, an important exception to Weber-Fechner's law is high-lighted: pain perception. One possible reason for the fact that pain perception rises in an exponential manner with the stimulus (multiple receptors and extracellular signalling) is presented in a separate spreadsheet. Again, the limits of the model are pointed out: thus, the model does not incorporate central processing.

**Laterale Hemmung im vereinfachten Retina Modell**

**Verstärkung von Amplitude & Kontrast**

Spalt oder Gitter  
Lichtintensität (variabel)

Photorezeptor  
Bipolarzellen:  
OFF  
ON

Verstärkungsfaktor (variabel)  
ON -3 (negativ)  
OFF 1 (positiv)

Ganglionzelle: integriert Signale  
n. Optikus  
...Wahrnehmung...

Finally, a classical model of lateral inhibition is presented in spreadsheet form and discussed with regard to Hermann's grid.

<http://www.leinroden.de/files/Hermannsgitter%20und%20die%20Folgen.pdf>

### Conclusions

Subjective estimates of staining intensity closely follow Weber-Fechner's law and corresponding experiments can be easily performed in the undergraduate laboratory. Simultaneously, students can be introduced to current methods of data evaluation via predefined spreadsheets of standard Excel software.